Logic: Propositional Tree Rules (Handout)

Here A and B can be any formula you like (e.g., A might be the formula (P&Q) → R).

Double Negation (\(\neg\neg\neg\neg\))

\[\neg\neg\neg\neg A\]
\[\neg\neg A \quad | \quad A\]

Conjunction (&)          Negated Conjunction (\(\neg\&\))

\[A \& B \quad \neg(A \& B)\]
\[\neg\quad / \quad \neg\]
\[A \quad \neg A \quad B \quad \neg B\]

Disjunction (\(\vee\))        Negated Disjunction (\(\neg\vee\))

\[A \vee B \quad \neg(A \vee B)\]
\[\neg\quad / \quad \neg\]
\[A \quad B \quad \neg A \quad \neg B\]

Conditional (\(\rightarrow\))        Negated Conditional (\(\neg\rightarrow\))

\[A \rightarrow B \quad \neg(A \rightarrow B)\]
\[\neg\quad / \quad \neg\]
\[\neg A \quad B \quad A \quad \neg B\]

Biconditional (\(\leftrightarrow\))        Negated Biconditional (\(\neg\leftrightarrow\))

\[A \leftrightarrow B \quad \neg(A \leftrightarrow B)\]
\[\neg\quad / \quad \neg\]
\[A \quad \neg A \quad A \quad \neg A\]
\[B \quad \neg B \quad \neg B \quad B\]

Using these nine simple “algorithmic” rules you can:

(i) **prove** any **valid** sequent;

(ii) construct a **counter-example** for any **invalid** sequent.
Flow Diagram for Constructing Truth Trees

(Step 1) List the elements of the initial list, adding a line number for each.

\[\text{⇓}\]

⇒ (Step 2) Pick a formula in an open branch and apply one of the tree rules and write down the result, adding \textit{lines numbers} and \textit{annotations}.

\[\text{⇑} \quad \text{⇓}\]

(Step 3) Put a tick (✓) next to the formula used.

\[\text{⇑} \quad \text{⇓}\]

(Step 4) Examine any branch. If it contains a formula A and its negation \(\neg A\), then place -X- after the final formula in the branch. That branch is \textit{closed}.

\[\text{⇑} \quad \text{⇓}\]

(Step 5) If every branch is closed, then the tree is \textbf{CLOSED. STOP}

If some branch is not closed, …

\[\text{⇑} \quad \text{⇓}\]

(Step 6) if every formula in every branch has been used and some branch is still open, then the tree is \textbf{FINISHED} and \textbf{OPEN}. Then \textbf{STOP}.

If some formula has not been used, …. 

\[\text{⇑} \quad \text{⇓}\]

(Step 7) go back to (Step 2)

\[\text{⇐}\]